TIPS & TRICKS
Class-XII
Subject: MATHEMATICS
Chapter Name : Integral Calculus (Chap : 7)

INDEFINITE INTEGRAL :

1. $\int \{f(x)\}^n f'(x)dx = \frac{\{f(x)\}^{n+1}}{n+1} + c$

2. $\int \frac{f'(x)}{f(x)} dx = \log_e |f(x)| + c$

3. $\int e^x [f(x) + f'9x)] = e^x f(x) + c.$ & $\int [f(x) + xf'(x)] dx = xf(x) + c$

4. $\int f(x)dx = F(x) + c \Rightarrow \int f(ax+b)dx = \frac{1}{a} F(ax+b) + c \, ; a \neq 0.$

5. $\int \frac{dx}{\sqrt{a^2+x^2}}$ or $\sqrt{a^2-x^2}\, dx$, put $x = a \tan\theta.$ or $x = a \cot\theta$.

6. $\int \frac{dx}{\sqrt{a^2-x^2}}$ or $\sqrt{a^2-x^2}\, dx$, put $x = a \sin\theta.$ or $x = a \cos\theta$.

7. $\int \frac{dx}{\sqrt{x^2-a^2}}$ or $\sqrt{x^2-a^2}\, dx$, put $x = a \sec\theta.$ or $x = a \operatorname{cosec}\theta$.

8. $\int \sqrt{\frac{a-x}{a+x}}\, dx$, put $x = a \cos 2\theta$.

9. $\int \sqrt{\frac{a-\alpha}{\beta-x}}\, dx$ or $\int \sqrt{(x-\alpha)(\beta-x)}\, dx$, put $x = \alpha \cos^2\theta + \beta \sin^2\theta.$

10. $\int \sqrt{\frac{a-\alpha}{x-\beta}}\, dx$ or $\int \sqrt{(x-\alpha)(x-\beta)}\, dx$, put $x = \alpha \sec^2\theta - \beta \tan^2\theta.$

11. $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, put $(x - \alpha) = t^2$ or $(x - \beta) = t^2$

12. Integration By Parts :

$$\int uv\, dx = u \int v dx - \int \left[u' \int v dv \right] dx$$

use I L A T E RULE :-

$I \rightarrow$ Inverse function, $L \rightarrow$ logarithmic function

$A \rightarrow$ Algebraic function, T – trigonometric function.

$E \rightarrow$ Exponential function.

13. $\int \sin^m x \cos^n x\, dx$

Case I : m, $n \in N \rightarrow$
- $\rightarrow$ m, n both are odd substitute either of the term.
- m, n both are even convert them multiple angles.
- $\rightarrow$ one of them is odd, substitute the term of even power.

Case II : m, $n \in Q$ such that $(m + n)$ = negative even integer substitute $\tan x = t$.

14. $\int \frac{dx}{a + b \sin^2 x}$ or $\int \frac{dx}{a + b \cos^2 x}$ or $\int \frac{dx}{a \sin^2 x + b \sin x . \cos x + c \cos^2 x} \longrightarrow$ put $\tan x = t$.

15. $\int \frac{dx}{a+b\sin x}$ or $\int \frac{dx}{a+b\cos x}$ or $\int \frac{dx}{a\sin x+b\sin x.\cos x+c\cos x}$

$\longrightarrow$ put $\tan\left(\frac{x}{2}\right)=t$ & $\sin x=\frac{2t}{1+t^2}, \cos x=\frac{1-t^2}{1+t^2}$

$$x=2\tan^{-1}t\,;dx=\frac{2dt}{1+t^2}$$

16. $\int \frac{a\cos x+b\sin x+c}{p\cos x+q\sin x+r}dx$

$N^r = l(D^r) + m(D^r)' + n.$

17. $\frac{p(x)}{q(x)}$ where p(x) & q(x) are polynomials such that degree of p(x) ≥ degree of q(x).

$\Rightarrow \frac{p(x)}{q(x)}=t(x)+\frac{p_1(x)}{q(x)}$ where degree of $p_1(x)$ is less than degree of q(x).

18. $\int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \sqrt{ax^2+bx+c}\,dx$

Express $ax^2 + bx + c$ in the form of perfect square.

19. $\int \frac{px+q}{ax^2+bx+c}dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

Express $px + q = l\,(D^r)' + m$

20. $\int \frac{(x^2+1)\,dx}{x^4+kx^2+1}$ or $\int \frac{(x^2-1)dx}{x^4+kx^2+1}$, dividing N^r and D^r by x^2.

21. $\int \frac{dx}{(ax+b)\sqrt{px+q}}$ & $\int \frac{dx}{(ax^2+bx+c)\sqrt{px+q}}$, put $px + q = t^2$.

22. $\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}}$, put $ax + b = \frac{1}{t}$ & $\int \frac{dx}{(ax^2+b)\sqrt{px^2+q}}$ put $x=\frac{1}{t}$.

23. $\int \frac{dx}{x(x^n+1)}$, $n\in N$ take x^n common & put $1 + x^{-n} = t$

DEFINITE INTEGRALS :

1. The fundamental theorem of calculus (Part 1) : $g(x)=\int_a^x f(t)\,dt,\ a\le x\le b\,; g'(x)=f(x)$

   The fundamental theorem of calculus (Part 2) : $\int_a^b f(x)dx=F(b)-F(a),\quad F'(x)=f(x)$

2. $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$, if f is piecewise continuous in (a, b).

3. $\int_{-a}^{a} f(x)dx=\int_0^a[f(x)+f(-x)]dx=\begin{cases}0\,; \text{ if } f(x) \text{ is an odd function}\\ 2\int_0^a f(x)dx\,; \text{ if } f(x) \text{ is an even function}\end{cases}$

4. $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

   $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

5. $\int_0^{2a} f(x)dx=\int_0^a f(x)dx+\int_0^a f(2a-x)\,dx=\begin{cases}2\int_0^a f(x)dx & \text{if } f(2a-x)=f(x)\\ 0 & \text{if } f(2a-x)=-f(x)\end{cases}$

6. $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$; $(n \in I)$ and $f(T+x) = f(x)$

& $\int_x^{T+x} f(t)dt = \int_0^{T} f(t)dt$

7. $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$; $f(T+x) = f(x),\ n \in I$

8. $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^{T} f(x)dx$; $f(T+x) = f(x), m, n \in I.$

9. Wallis formula :

(i) $\int_0^{\frac{\pi}{2}} \sin^n x dx = \int_0^{\frac{\pi}{2}} \cos^n x dx = \frac{(n-1)(n-3)(n-5)......(1 \text{ or } 2)}{n(n-2)(n-4)......(1 \text{ or } 2)} k$

where $k = \begin{cases} \frac{\pi}{2} \text{ if n is even} \\ 1 \text{ if n is odd.} \end{cases}$

(ii) $\int_0^{\frac{\pi}{2}} \sin^n x . \cos^m x = \frac{[(n-1)(n-3)(n-5)....1 \text{ or } 2][(m-1)(m-3)(m-5)...1 \text{ or } 2]}{(m+n)(m+n-2)(m-n-4)...1 \text{ or } 2} k$

where $k = \begin{cases} \frac{\pi}{2} \text{ if n is even} \\ 1 \text{ if n is odd.} \end{cases}$

10. $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt = f(h(x))h'(x) - f(g(x)).g'(x)$. [Newton Leibnitz Formula]

11. $\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=0}^{n-1} f(a+rh)$; $h = \frac{b-a}{n}$

$\frac{r}{n} \equiv x$ Lower limit = $\lim_{n\to\infty} \frac{r}{n}$ (put r = 0) (ie. initial value of r)

$\frac{1}{n} \equiv dx$

$\lim_{n\to\infty} \sum \equiv \int$ Upper limit = $\lim_{n\to\infty} \frac{r}{n}$ (put r = n – 1) (ie. final value of r)

12. If $f(x) \in [m, M]$ then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

13. If $f(x) \le \phi(x)$ for $a \le x \le b$ then

$\int_a^b f(x)dx \le \int_a^b \phi(x)dx$

14. $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\, dx$

15. If $f(x) \ge 0$ in [a, b] then $\int_a^b f(x)\, dx \ge 0$